

# Nonlinear Control of a Twin-Lift Helicopter Configuration

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Significant pilot work load involved in flying a twin-lift helicopter configuration necessitates the development of highly augmented flight control systems for satisfactory performance. Two nonlinear control philosophies based on feedback linearization are advanced for this configuration. The controller performance, together with the sensitivity to a few parameter variations, is studied in a nonlinear simulation. Controller implementation aspects are discussed.

## Nomenclature

$e_1, e_2$	= horizontal and vertical payload position errors
$g$	= acceleration due to gravity
$H$	= tether length
$K_i$	= integral feedback gain
$K_p$	= proportional feedback gain
$K_r$	= rate feedback gain
$M$	= helicopter mass
$M_L$	= payload mass
$T$	= helicopter main rotor thrust
$U$	= pseudocontrol variable
$Y$	= component of the helicopter main rotor thrust in the horizontal direction
$Y_a$	= aerodynamic force on the helicopter in the horizontal direction
$Y_{aL}$	= aerodynamic force on the payload in the horizontal direction
$y$	= horizontal position of the helicopter with respect to a chosen inertial frame
$y_L$	= horizontal position of the payload with respect to a chosen inertial frame
$Z$	= component of the helicopter main rotor thrust in the vertical direction
$Z_a$	= aerodynamic force on the helicopter in the vertical direction
$Z_{aL}$	= aerodynamic force on the payload in the vertical direction
$z$	= vertical position of the helicopter with respect to the chosen inertial frame
$z_L$	= vertical position of the payload with respect to the chosen inertial frame
$\alpha, \beta$	= intermediate variables in control law calculations
$\gamma, \delta$	= intermediate variables in control law calculations
$\Delta y$	= horizontal separation between the two helicopters
$\Delta z$	= vertical separation between the two helicopters
$\phi$	= helicopter roll attitude

## Subscripts

1	= variables associated with the first helicopter
2	= variables associated with the second helicopter
c	= commanded variables
L	= payload related variables
.	= differentiation with respect to time

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## Introduction

THE twin-lift concept has been in existence in the helicopter industry for more than three decades.<sup>1</sup> The primary motivation here is to improve the helicopter payload capacity while avoiding the need for building larger and larger vehicles. It has been shown that beyond a certain payload capacity, the economics of the helicopter performance obeys the law of diminishing returns.<sup>2</sup> Several twin-lift configurations have been proposed in the literature.<sup>3</sup> Of these, the present study focuses on the pendant configuration. In its most general form, the pendant configuration consists of two dissimilar helicopters carrying a payload suspended between them using tethers of unequal length. Emergency payload disconnect devices are incorporated in the system to ensure crew safety. A spreader bar is sometimes used to relieve the helicopters of having to bias their pitch-roll controls for maintaining the separation between the rotor disks while carrying payloads of relatively smaller dimensions. Additionally, the spreader bar configuration permits zero trim roll and pitch attitudes while the helicopters are in hover. The disadvantages of using a spreader bar are that it introduces additional dynamics including an unstable mode in hover<sup>2</sup> and that it is inconvenient from a logistics point of view. These difficulties become even more pronounced while examining the multilift configuration. In view of these issues, the present research will focus on the study of a twin-lift configuration without the spreader bar.

An earlier study<sup>2</sup> dealt with the dynamical properties of the linearized twin-lift system about steady-state hover conditions. That work also described the synthesis of a stability augmentation system using the linearized vehicle model. Manual control aspects of such a twin-lift configuration about the hover flight condition has been studied in considerable detail in Ref. 4. The pilot opinion of the flying qualities of the twin-lift configuration in a completely manual control mode has not been favorable, primarily due to the significant increase in cockpit work load. Since this factor adversely affects the speed of executing various twin-lift tasks, a high degree of

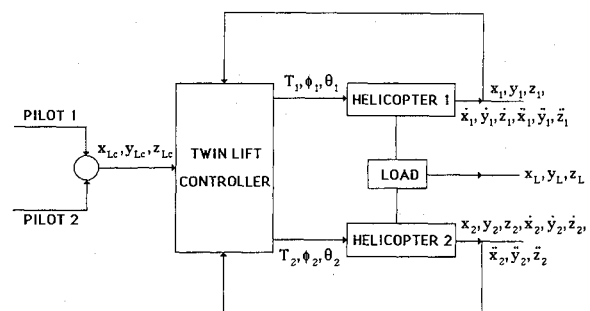


Fig. 1 Twin-lift control system.

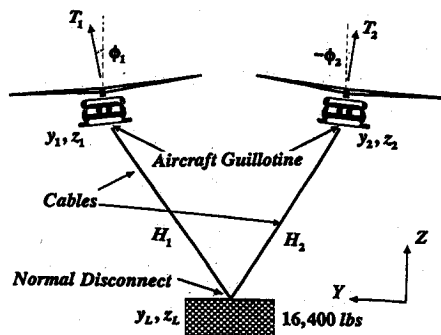


Fig. 2 Twin-lift configuration.

stability and command augmentation has been deemed essential<sup>4</sup> for a safe and satisfactory operation.

Two distinct operational modes may be identified while synthesizing an autopilot for this system. First, it is possible to conceive of a completely automatic mode wherein the pilot sets up the desired final conditions and the flight control computer then flies the twin-lift configuration automatically using an internally generated trajectory. This mode of operation may be desirable for missions such as takeoff and landing under poor visibility conditions. In the second operational mode, one or both of the pilots use three axis control columns to continuously command the payload position. The twin-lift autopilot then tracks these commands in real time while maintaining adequate separation between the helicopters. Note that it is possible to provide control columns in both helicopter cockpits, permitting the pilots to fly the configuration in tandem, much like the modern day transport aircraft. The twin-lift autopilot proposed in this paper is valid for both of these operational modes. A block diagram of the twin lift autopilot is given in Fig. 1.

The schematic of a twin-lift configuration in the transversal plane is given in Fig. 2. This figure also defines some of the variables used in the ensuing development. As envisaged in the present research, the twin-lift controller interprets the pilot commands, collects sensor information from the two helicopters, and generates control commands for individual helicopters. The objective here is to maintain the separation between the two vehicles while tracking the commanded payload position. The twin-lift control law commands the helicopter pitch, yaw and roll attitudes, together with the main rotor thrust for the two helicopters. Existing flight control systems onboard the two helicopters have the joint responsibility of tracking these commanded values using longitudinal cyclic, lateral cyclic, and collective control settings. The present research focus will be on the autopilot synthesis for the twin-lift system. The emphasis will be on providing a high degree of command and stability augmentation permitting operations with a moderate to low degree of pilot involvement.

Although control system design based on linearized vehicle models as in Ref. 2 is certainly adequate for maintaining steady-state flight conditions, that approach will entail gain scheduling to obtain satisfactory dynamic response during maneuvers requiring the full control excursions. This is due to the fact that the twin-lift system is highly nonlinear and has to be handled as such if this system capabilities are to be fully exploited. The control laws presented in this paper are based on feedback linearization<sup>5,6</sup> and explicitly use the system nonlinearities in the feedback loop.

Two control laws will be advanced for the twin-lift configuration. In the first control philosophy, each helicopter is charged with the task of controlling specific sets of output variables. For instance, one of the helicopters may be assigned the responsibility for payload positioning, while the other helicopter maintains the lateral separation between the two vehicles. For the lack of better terminology, previous literature<sup>2,4</sup> refers to this as the master-slave control concept. In the present work, this control philosophy will be termed the role-

assigned control law. In the second control concept, both helicopters are jointly responsible for the control of all of the flight variables. This scheme will be termed the cooperative control concept. The performance, advantages, and disadvantages of both these control concepts will be examined in the following.

The analysis presented here will be limited to the study of the twin-lift system dynamics in the transversal plane. Extension to the three-dimensional case is not difficult, although nontrivial. This aspect will not be pursued in the present paper.

### Nonlinear Model of the Twin-Lift System

Figure 2 schematically illustrates the twin-lift configuration together with the coordinate system and a definition of variables. Depending upon the degrees of freedom considered, rather sophisticated mathematical models can be constructed for this system.<sup>3</sup> For the present study, however, a point-mass approximation will be used. In this approximation, the helicopter attitude control systems are assumed to have much larger bandwidths when compared with the highest frequency component in the attitude commands generated by the twin-lift control law. In this case, the helicopter attitudes can be treated as "control-like" in the outer-loop control law synthesis. The validity of this assumption is further reinforced if the tether attachment point on the helicopters are close to their center of gravity. In this case, the payload exerts negligible moments on the airframe and the helicopter attitude dynamics is not directly affected by the payload motion. If desired, these control laws can be corrected for neglected attitude dynamics by formally invoking singular perturbation theory.<sup>7</sup> Note that

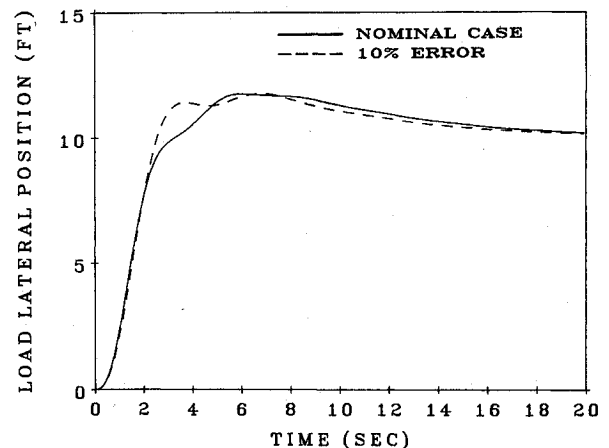


Fig. 3 Response of the role-assigned controller to a 10-ft step lateral motion command.

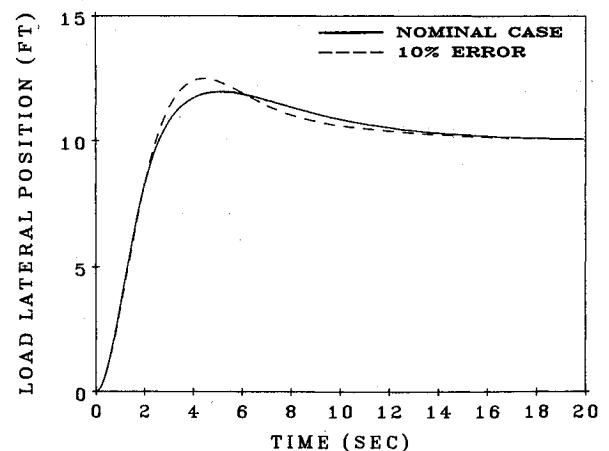


Fig. 4 Response of the cooperative controller to a 10-ft step lateral motion command.

such a separation of the attitude dynamics from the point-mass motion is meaningful in the twin-lift control law synthesis since the helicopters must be able to function independently when disconnected from the twin-lift configuration.

The dynamical model of the twin-lift configuration in the transversal plane can be obtained by considering the force equilibrium along the vertical and horizontal directions. This yields

$$Z_{a1} + Z_{a2} + Z_{aL} + T_1 \cos \phi_1 + T_2 \cos \phi_2 - M_1(\ddot{z}_1 + g) - M_L(\ddot{z}_L + g) - M_2(\ddot{z}_2 + g) = 0 \quad (1)$$

$$Y_{a1} + Y_{a2} + Y_{aL} + T_1 \sin \phi_1 + T_2 \sin \phi_2 - M_1\ddot{y}_1 - M_L\ddot{y}_L - M_2\ddot{y}_2 = 0 \quad (2)$$

The aerodynamic forces on the helicopters and the payload  $Y_{a1}$ ,  $Z_{a1}$ ,  $Y_{a2}$ ,  $Z_{a2}$ ,  $Y_{aL}$ , and  $Z_{aL}$  depend on the helicopter configuration, the payload shape, altitude, and the velocity components. Any desired degree of sophistication in the computation of these quantities can be included in the ensuing analysis. The control variables in this model are the helicopter rotor thrusts  $T_1$  and  $T_2$  and the roll attitudes  $\phi_1$  and  $\phi_2$ . The assumptions invoked in deriving Eqs. (1) and (2) are that the tethers are rigid and massless with negligible aerodynamic forces and that the pin joints are frictionless.

Since the payload attachment point is a pin joint and the tethers are massless rigid links, the moments about this point should vanish. This fact produces the two following equations:

$$(T_1 \sin \phi_1 + Y_{a1} - M_1\ddot{y}_1)(z_1 - z_L) + (M_1g + M_1\ddot{z}_1 - T_1 \cos \phi_1 - Z_{a1})(y_1 - y_L) = 0 \quad (3)$$

$$(T_2 \sin \phi_2 + Y_{a2} - M_2\ddot{y}_2)(z_2 - z_L) + (M_2g + M_2\ddot{z}_2 - T_2 \cos \phi_2 - Z_{a2})(y_2 - y_L) = 0 \quad (4)$$

The kinematic constraints in the problem are that the tether lengths  $H_1$  and  $H_2$  are fixed, i.e.,

$$(y_1 - y_L)^2 + (z_1 - z_L)^2 = H_1^2 = \text{const} \quad (5)$$

$$(y_2 - y_L)^2 + (z_2 - z_L)^2 = H_2^2 = \text{const} \quad (6)$$

If desired, Eqs. (5) and (6) may be differentiated twice with respect to time to obtain  $\ddot{y}_L$  and  $\ddot{z}_L$  in terms of helicopter acceleration components. These may be substituted back into Eqs. (1-4) to obtain four implicit second-order nonlinear dif-

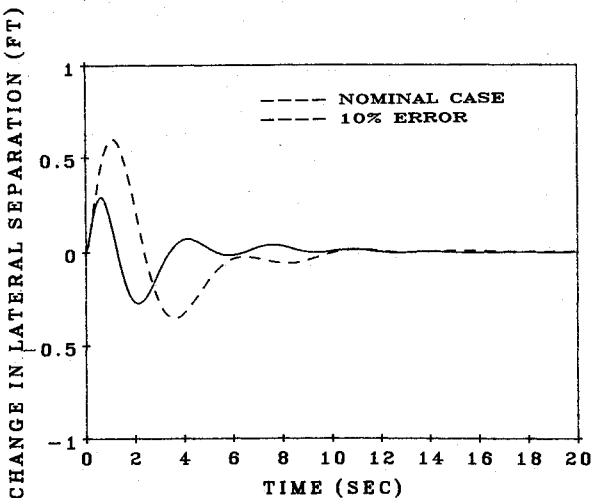


Fig. 5 Role-assigned controller: variation in lateral separation in response to a simultaneous 10-ft step vertical and lateral motion command.

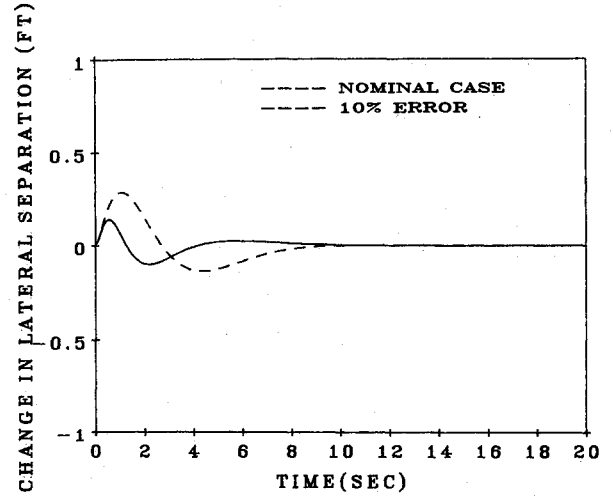


Fig. 6 Cooperative controller: variation in lateral separation in response to a simultaneous 10-ft step vertical and lateral motion command.

ferential equations for the helicopter position components  $y_1$ ,  $z_1$ ,  $y_2$ ,  $z_2$ . Equations (5) and (6) may also be used to verify the simulation accuracy by comparing the tether lengths  $H_1$  and  $H_2$  with the left-hand sides during the integration of the equations of motion.

However, an alternate set of variables is more appropriate for the present problem. Noting that the variables to be controlled are the payload position components  $y_L$  and  $z_L$  and the separation between the helicopters  $(y_1 - y_2)$  and  $(z_1 - z_2)$ , the equations of motion may be formulated in terms of these variables. Significant saving in algebra results by recognizing the fact that the helicopter acceleration components  $\ddot{y}_1$ ,  $\ddot{y}_2$ ,  $\ddot{z}_1$ , and  $\ddot{z}_2$  are available from onboard instruments. Thus, one has

$$\ddot{y}_L = (1/M_L)[Y_{a1} + Y_{a2} + Y_{aL} + T_1 \sin \phi_1 + T_2 \sin \phi_2 - M_1\ddot{y}_1 - M_2\ddot{y}_2] \quad (7)$$

$$\ddot{z}_L = (1/M_L)[Z_{a1} + Z_{a2} + Z_{aL} + T_1 \cos \phi_1 + T_2 \cos \phi_2 - M_1(\ddot{z}_1 + g) - M_2(\ddot{z}_2 + g) - M_Lg] \quad (8)$$

$$\Delta \ddot{z} = (1/M_1)\{T_1 \cos \phi_1 + Z_{a1} - M_1g - (T_1 \sin \phi_1 - M_1\ddot{y}_1 + Y_{a1})[(z_1 - z_L)/(y_1 - y_L)]\} - (1/M_2)\{T_2 \cos \phi_2 + Z_{a2} - M_2g - (T_2 \sin \phi_2 - M_2\ddot{y}_2 + Y_{a2})[(z_2 - z_L)/(y_2 - y_L)]\} \quad (9)$$

$$\Delta \ddot{y} = (1/M_1)\{T_1 \sin \phi_1 + Y_{a1} + (M_1g + M_1\ddot{z}_1 - T_1 \cos \phi_1 - Z_{a1})[(y_1 - y_L)/(z_1 - z_L)]\} - (1/M_2)\{T_2 \sin \phi_2 + Y_{a2} + (M_2g + M_2\ddot{z}_2 - T_2 \cos \phi_2 - Z_{a2})[(y_2 - y_L)/(z_2 - z_L)]\} \quad (10)$$

where

$$\Delta y = y_1 - y_2, \quad \Delta z = z_1 - z_2 \quad (11)$$

The four second-order nonlinear differential Eqs. (7-10) may be used for the twin-lift controller synthesis.

### Feedback Linearization and Control System Design

The objectives of the twin-lift controller are to maintain the horizontal and vertical separations  $\Delta y$  and  $\Delta z$  at commanded values and to move the payload from its initial position to the

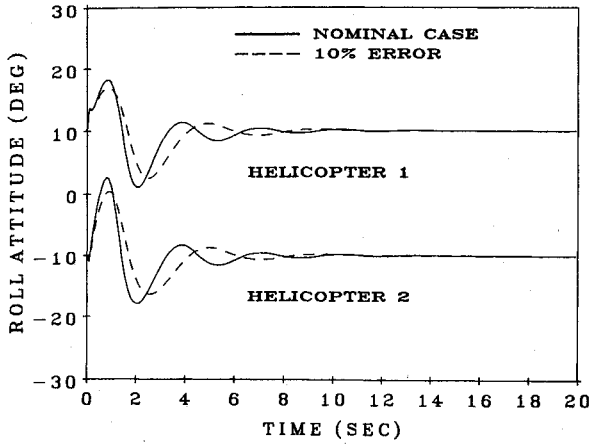


Fig. 7 Roll attitude evolution during a simultaneous 10-ft step vertical and lateral motion command for the role-assigned controller.

final position in a specified time period. The four control variables in the model, viz., the rotor thrusts  $T_1$  and  $T_2$  and the roll attitudes  $\phi_1$  and  $\phi_2$ , will be used to control the four quantities  $y_L$ ,  $z_L$ ,  $\Delta y$ , and  $\Delta z$ .

The model given by Eqs. (7-10) is in a form suitable for feedback linearization as in Refs. 5 and 6. Additionally, as in Ref. 8, it is feasible to include sensor and actuator nonlinearities such as saturation, hysteresis, and dead zone in the analysis. Denoting the right-hand sides of the system of Eqs. (7-10) by four *pseudocontrol variables*<sup>6</sup>  $U_1$ ,  $U_2$ ,  $U_3$ , and  $U_4$  results in a linear time-invariant system:

$$\ddot{y}_L = U_1, \quad \ddot{z}_L = U_2 \quad (12)$$

$$\Delta \ddot{z} = U_3, \quad \Delta \ddot{y} = U_4 \quad (13)$$

The control system design for these four decoupled linear time-invariant systems may be accomplished using the classical frequency-domain approach.

Since the variables  $\Delta y$  and  $\Delta z$  need to be regulated about their set point values, a proportional-plus-derivative feedback is adequate.<sup>9</sup> This conclusion is based on the system order, the expected nature of commands, and the permissible steady-state error. Thus, the control law for the lateral and vertical separation are of the form

$$U_3 = K_{p2} (\Delta z_c - \Delta z) - K_{r2} \Delta \dot{z} \quad (14)$$

$$U_4 = K_{p1} (\Delta y_c - \Delta y) - K_{r1} \Delta \dot{y} \quad (15)$$

In Eqs. (14) and (15),  $\Delta y_c$  and  $\Delta z_c$  are the commanded horizontal and vertical separations, respectively. The desired horizontal separation is typically chosen to be about twice the rotor diameter. In the case where the tether lengths are equal, the vertical separation between the vehicles is normally maintained at zero. Under this feedback law, the control loops for  $\Delta y$  and  $\Delta z$  will have a spring-mass-damper type of dynamic behavior. The proportional feedback gain  $K_p$  and the rate feedback gain  $K_r$  determine the system natural frequency  $\omega_n$  and damping  $\zeta$  as

$$K_p = \omega_n^2, \quad K_r = 2\zeta\omega_n \quad (16)$$

It is desirable to have an adequately damped response from the system without excessive overshoot. This fixes the damping ratio at about 0.7. The natural frequency selection is based on the bandwidth of the individual helicopter flight control systems. Typically, the twin-lift control law bandwidth is chosen to be about one-fourth that of the helicopter flight control

system bandwidth. It is not difficult to verify that these control loops will have a zero steady-state error for step inputs.

Since precise trajectory control is desired for the payload position components  $y_L$  and  $z_L$ , a proportional-plus-integral-plus-derivative control law will be employed for these variables. The development of tracking control laws for the payload position components proceed as follows. Define the tracking error as the state variables in these channels as

$$e_1 = y_{Lc} - y_L \quad (17a)$$

$$e_2 = z_{Lc} - z_L \quad (17b)$$

Differentiating expression (17) twice with respect to time and using expression (12), the equations for the error dynamics turn out to be

$$\ddot{e}_1 = \ddot{y}_{Lc} - U_1 \quad (18)$$

$$\ddot{e}_2 = \ddot{z}_{Lc} - U_2 \quad (19)$$

A proportional-integral-derivative control law is next designed for regulating the error states about zero. These control laws are of the form

$$U_1 = \ddot{y}_{Lc} + K_{p3} e_1 + K_{r3} \dot{e}_1 + K_{i3} \int_0^t e_1 dt \quad (20)$$

$$U_2 = \ddot{z}_{Lc} + K_{p4} e_2 + K_{r4} \dot{e}_2 + K_{i4} \int_0^t e_2 dt \quad (21)$$

Unlike the previous case, relating the feedback gains to the system time response specifications is not direct. However, it is still possible to use classical control synthesis tools for carrying out the system design. In the present case, the design was accomplished using the root-locus method. Alternate synthesis techniques such as the linear-quadratic-regulator approach<sup>10</sup> can also be used to determine the feedback gains.

Once the control system design using the feedback linearized model is complete, the real control variables  $T_1$ ,  $T_2$ ,  $\phi_1$ , and  $\phi_2$  need to be recovered from the pseudocontrol variables  $U_1$ ,  $U_2$ ,  $U_3$ , and  $U_4$ . To this end, denote

$$\begin{aligned} Y_1 &= T_1 \sin \phi_1, & Z_1 &= T_1 \cos \phi_1 \\ Y_2 &= T_2 \sin \phi_2, & Z_2 &= T_2 \cos \phi_2 \end{aligned} \quad (22)$$

Next, using the fact that  $U_1$ ,  $U_2$ ,  $U_3$ , and  $U_4$  can be computed using the feedback control laws of Eqs. (14), (15), (20),

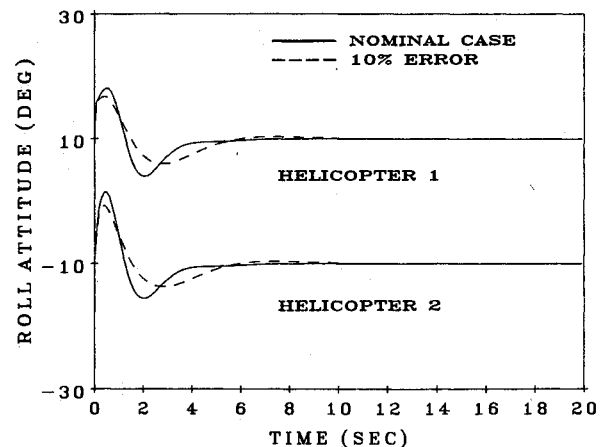


Fig. 8 Roll attitude evolution during a simultaneous 10-ft step vertical and lateral motion command for the cooperative controller.

and (21), Eqs. (7-10) may be expressed in a matrix form as

$$\begin{bmatrix} M_L U_1 + \alpha \\ U_3 + \beta \\ M_L U_2 + \gamma \\ U_4 + \delta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ -\frac{(z_1 - z_L)}{M_1(y_1 - y_L)} & \frac{1}{M_1} & \frac{(z_2 - z_L)}{M_2(y_2 - y_L)} & -\frac{1}{M_2} \\ 0 & 1 & 0 & 1 \\ \frac{1}{M_1} & -\frac{(y_1 - y_L)}{M_1(z_1 - z_L)} & -\frac{1}{M_2} & \frac{(y_2 - y_L)}{M_2(z_2 - z_L)} \end{bmatrix} \begin{bmatrix} Y_1 \\ Z_1 \\ Y_2 \\ Z_2 \end{bmatrix} \quad (23)$$

where

$$\alpha = M_1 \ddot{y}_1 + M_2 \ddot{y}_2 - Y_{a1} - Y_{a2} - Y_{aL} \quad (24)$$

$$\begin{aligned} \beta = & -(1/M_1)\{Z_{a1} - M_1 g \\ & - (Y_{a1} - M_1 \ddot{y}_1)[(z_1 - z_L)/(y_1 - y_L)]\} \\ & + (1/M_2)\{Z_{a2} - M_2 g \\ & - (Y_{a2} - M_2 \ddot{y}_2)[(z_2 - z_L)/(y_2 - y_L)]\} \end{aligned} \quad (25)$$

$$\gamma = M_1(\ddot{z}_1 + g) + M_2(\ddot{z}_2 + g) + M_L g - Z_{a1} - Z_{a2} - Z_{aL} \quad (26)$$

$$\begin{aligned} \delta = & -(1/M_1)\{Y_{a1} \\ & + (M_1 g + M_1 \ddot{z}_1 - Z_{a1})[(y_1 - y_L)/(z_1 - z_L)]\} \\ & + (1/M_2)\{Y_{a2} + (M_2 g + M_2 \ddot{z}_2 - Z_{a2})[(y_2 - y_L)/(z_2 - z_L)]\} \end{aligned} \quad (27)$$

The matrix Eq. (23) can be solved for the unknowns  $Y_1$ ,  $Y_2$ ,  $Z_1$ , and  $Z_2$ . Once these quantities are computed, the commanded rotor thrusts and roll attitudes for the two helicopters may be recovered as

$$\begin{aligned} \phi_1 &= \tan^{-1} \left[ \frac{Y_1}{Z_1} \right], \quad T_1 = \sqrt{Y_1^2 + Z_1^2} \\ \phi_2 &= \tan^{-1} \left[ \frac{Y_2}{Z_2} \right], \quad T_2 = \sqrt{Y_2^2 + Z_2^2} \end{aligned} \quad (28)$$

There are two distinct approaches for solving Eq. (23). The first of these is to invert the  $4 \times 4$  matrix on the right side, followed by the use of Eqs. (28) to obtain the commanded roll attitudes  $\phi_1$  and  $\phi_2$  and main rotor thrusts  $T_1$  and  $T_2$ . Such a solution produces what may be termed a cooperative control philosophy since both helicopter control laws monitor and control all of the commanded variables. On the other hand,

the control law computations can be significantly reduced by assigning roles to the helicopters. In this case, the output variables of interest are partitioned so that each helicopter is assigned the responsibility for controlling only a subset of the output variables. The individual helicopter controls are then computed by assuming that the other helicopter follows its commanded variables perfectly. This control philosophy is termed the role-assigned control in the present research. In this case, the matrix Eq. (23) can be partitioned and solved. For instance, one of the helicopters may be used for controlling the lateral payload position  $y_L$  and the vertical separation  $\Delta z$  while the second helicopter may be charged with the responsibility for controlling the vertical payload position  $z_L$  and the lateral separation  $\Delta y$ . In this case, the control law computations will involve the inversion of two  $2 \times 2$  matrices as follows:

$$\begin{bmatrix} M_L U_1 + \hat{\alpha} \\ U_3 + \hat{\beta} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{(z_1 - \Delta z - z_L)}{M_2(y_1 - \Delta y - y_L)} & \frac{1}{M_2} \end{bmatrix} \begin{bmatrix} \hat{Y}_2 \\ \hat{Z}_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -\frac{(z_1 - z_L)}{M_1(y_1 - y_L)} & \frac{1}{M_1} \end{bmatrix} \begin{bmatrix} Y_1 \\ Z_1 \end{bmatrix} \quad (29)$$

$$\begin{bmatrix} M_L U_2 + \hat{\gamma} \\ U_4 + \hat{\delta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{1}{M_1} & -\frac{(y_2 - \Delta y - y_L)}{M_1(z_2 - \Delta z - z_L)} \end{bmatrix} \begin{bmatrix} \hat{Y}_1 \\ \hat{Z}_1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -\frac{1}{M_2} & \frac{(y_2 - y_L)}{M_2(z_1 - z_L)} \end{bmatrix} \begin{bmatrix} Y_2 \\ Z_2 \end{bmatrix} \quad (30)$$

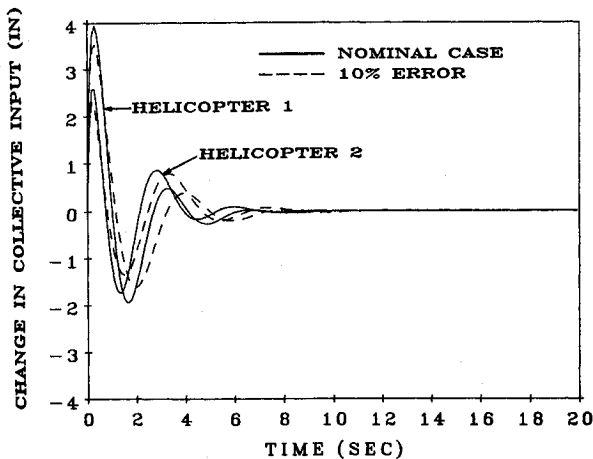


Fig. 9 Variation in collective input during a simultaneous 10-ft step vertical and lateral motion command for the role-assigned controller.

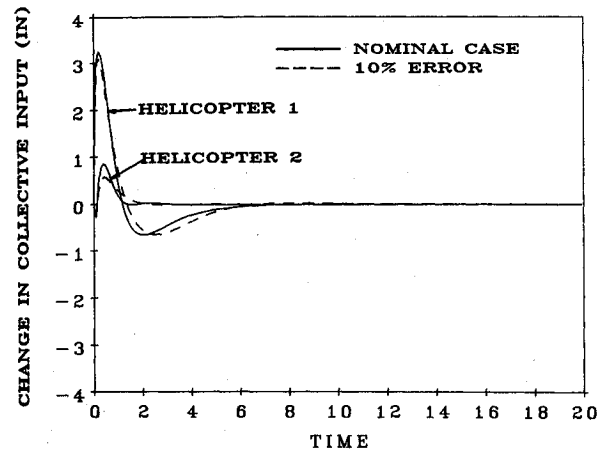


Fig. 10 Variation in collective input during a simultaneous 10-ft step vertical and lateral motion command for the cooperative controller.

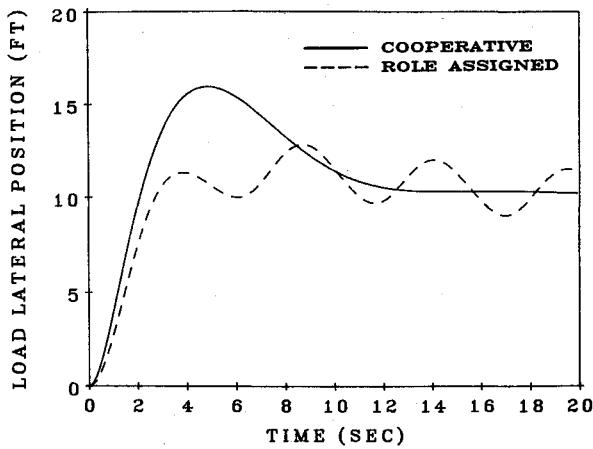


Fig. 11 Lateral response of the payload to a simultaneous 10-ft step vertical and lateral motion command in the presence of relative performance degradation.

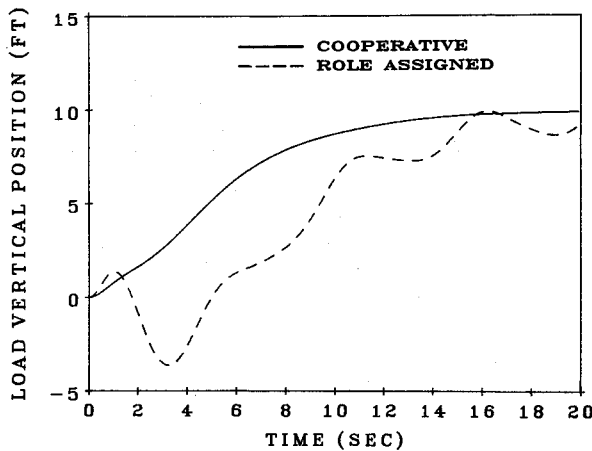


Fig. 12 Vertical response of the payload to simultaneous 10-ft vertical and lateral step motion command in the presence of relative performance degradation.

The variables denoted by “ $\sim$ ” are the quantities estimated based on the payload position commands. Note that alternate role assignments are also possible. In order to separate the twin-lift control law in this manner, it is necessary that each helicopter fulfills its role with tight tolerances. Otherwise, the helicopter control loops can interact adversely, producing an unacceptable system performance.

In addition to being computationally efficient, the role-assigned control philosophy is more easily generalized to the multilift case. For instance, in a planar multilift concept involving  $n$  helicopters, the dimension of the matrix to be inverted in the case of the cooperative control concept will be  $2n \times 2n$ , while the role-assigned control concept will still require only  $2 \times 2$  matrix inversions. Additionally, the computational work load is distributed in the role-assigned control concept.

Although the role-assigned control philosophy is an approximate one, under ideal conditions, this control configuration can be made to have satisfactory performance by appropriate command tailoring and feedback-gain selection. However, if one of the helicopters suffers partial performance degradation, the cooperative control concept would have a much more graceful failure mode. Indeed, this is the main point that justifies the use of the cooperative control concept despite the increased computational complexity.

In summary, the quantities required for implementing these control laws are the individual helicopter acceleration, velocity and position components, together with the aerodynamic

models. The payload position and velocity components are computed using the kinematics of the configuration.

### Controller Performance Results

The gains required for implementing the feedback linearized control laws of Eqs. (14), (15), (20), and (21) were first synthesized using the root-locus technique. For the UH-60 helicopter considered in the present study, the roll attitude control system has about 4 rad/s bandwidth, while the rotor thrust system bandwidth is approximately 30 rad/s. Based on this data, a proportional gain of 1.2, a derivative gain of 2, and an integral gain of 0.2 were used for the cooperative control law. The corresponding values for the role-assigned control law were a proportional gain of 1.5, derivative gain of 3.0, and an integral gain of 0.2. Since the UH-60 helicopter has a rotor radius of 26.83 ft, the horizontal separation between the helicopters was maintained at 107.3 ft, corresponding to twice the rotor diameter. The vertical separation between the two vehicles was maintained at zero. Assuming that a 10-deg trim roll attitude is acceptable to the helicopter pilot and a payload weight equal to the helicopter weight, the tether lengths were calculated to be 114.8 ft.

A nonlinear simulation of the twin-lift helicopter system is employed for evaluating the performance of the synthesized controllers. This simulation uses the UH-60 helicopter aerodynamic data from Ref. 11. The payload aerodynamic forces were not included in this simulation. The equations of motion are integrated using a second-order Runge-Kutta method.

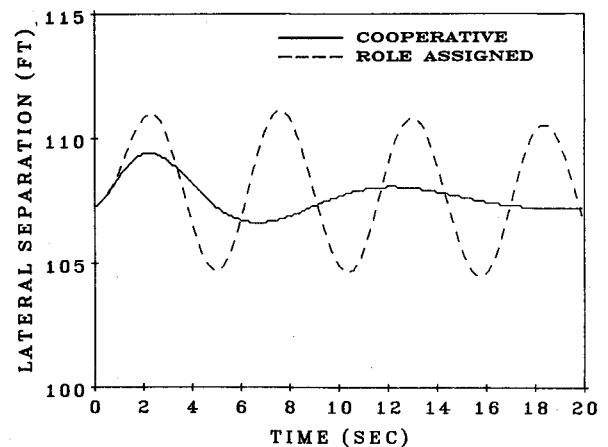


Fig. 13 Helicopter lateral separation response to a simultaneous 10-ft step vertical and lateral motion command in the presence of relative performance degradation.

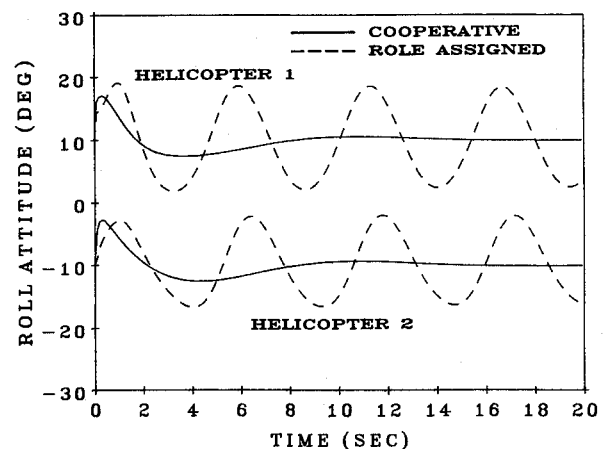


Fig. 14 Roll attitude evolution during a simultaneous 10-ft step vertical and lateral motion command in the presence of relative performance degradation.

Both the role-assigned control and the cooperative control schemes are implemented in the simulation.

The response of the role-assigned controller for a step lateral payload motion command is given in Fig. 3. The high-order nature of this response is clearly observable. The response of the cooperative control scheme for the same input is given in Fig. 4. The response is close to that of a second-order system subject to a step input. Figures 5 and 6 illustrate the variation in lateral separation between the two helicopters when the system is subject to a simultaneous step payload motion command of 10 ft in the vertical and lateral directions. From these figures, it can be seen that the system is able to maintain the separation within tight tolerances. The variation in helicopter collective deflection from the equilibrium and the roll attitudes corresponding to these responses are given in Figs. 7-10. The collective deflections are computed by using the helicopter aerodynamic model given in Ref. 11.

Next, the nonlinear controller is evaluated in the presence of modeling uncertainties. Since aerodynamic forces can be predicted only within 5-15% of their actual values,<sup>12</sup> the controller performance needs to be evaluated in the presence of these perturbations. Additionally, the influence of sensor errors such as bias and scale-factor uncertainties needs to be examined. Although several runs were made introducing various perturbations, the influence of main rotor performance variations and accelerometer scale-factor error will be illustrated in this section. To this end, the aerodynamic models in the nonlinear controller calculations and the accelerometer scale factors were perturbed by 10% in the negative direction.

The resulting changes in the system responses are indicated by dotted lines in Figs. 3-10. From these figures, it can be seen that the controller is relatively robust with respect to these parameter uncertainties.

The influence of relative helicopter performance degradation on the twin-lift control law performance is another issue of importance. For this study, the performance of one of the helicopters is degraded by 20% while maintaining the other helicopter parameters at their nominal value. The helicopter with a performance degradation is assumed to be able to achieve only 80% of the commanded rotor thrust and roll attitude. The lateral and vertical helicopter-payload responses when subjected to the step input are shown in Figs. 11 and 12. The response of the role-assigned controller can be seen to be oscillatory, while the cooperative controller produces an acceptable, although somewhat degraded, response. The lateral separation and roll attitude commands corresponding to Figs. 11 and 12 are given in Figs. 13 and 14. These results indicate that the cooperative control strategy may be superior to the role-assigned control concept in the presence of performance degradation in one of the helicopters.

Research is currently under way to extend the present work to include more sophisticated system models including three-dimensional helicopter-payload motion and a distributed-parameter tether model. Inclusion of swash plate hysteresis and saturation is also contemplated. Extension of these control techniques to the multilift case will be of future interest.

## Conclusions

The development of role-assigned control and cooperative control strategies for the automatic control of a twin-lift helicopter configuration was discussed in this paper. These schemes are based on feedback linearization and are useful for executing maneuvers requiring full control excursions. The sensitivity of these nonlinear control concepts to variations in the helicopter main rotor thrust and accelerometer scale factor was studied using a point-mass simulation. The influence of the performance degradation in one of the helicopters was also examined. The cooperative control concept was shown to be more robust than the role-assigned control concept in the presence of relative helicopter performance degradation.

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